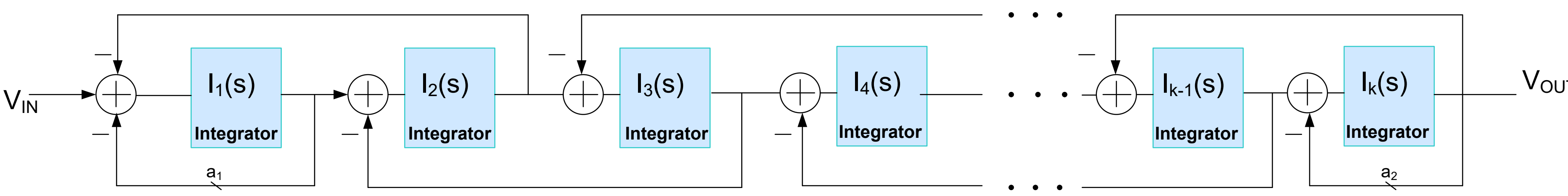
EE 508
Lecture 34

Leapfrog Networks
Transconductor Design

Review from last lecture

Leapfrog Filters

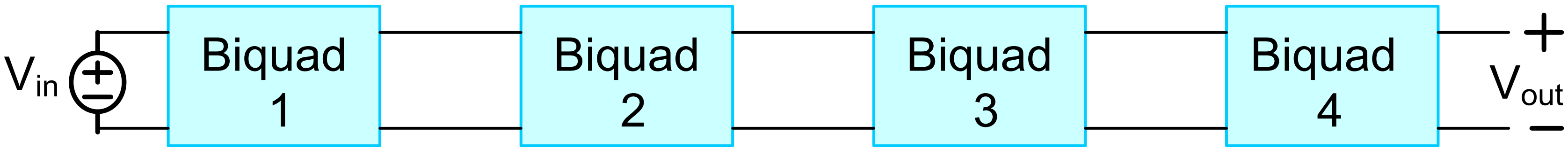


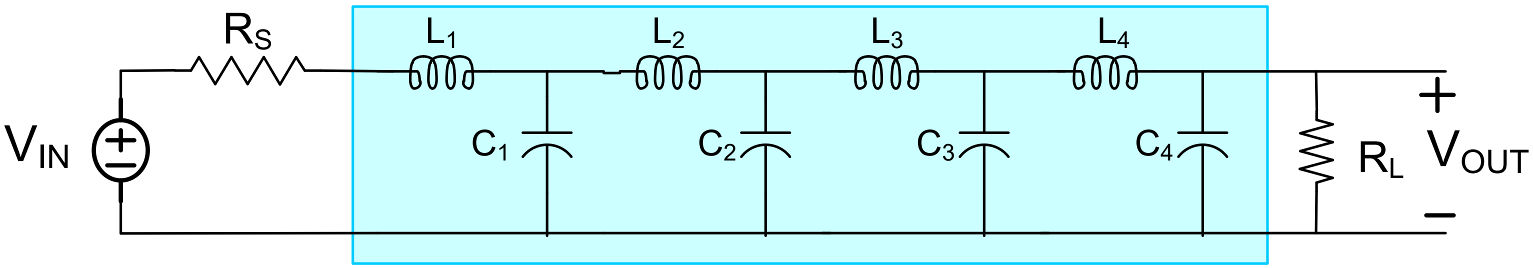
Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Review from last lecture

Implications of Theorem 1

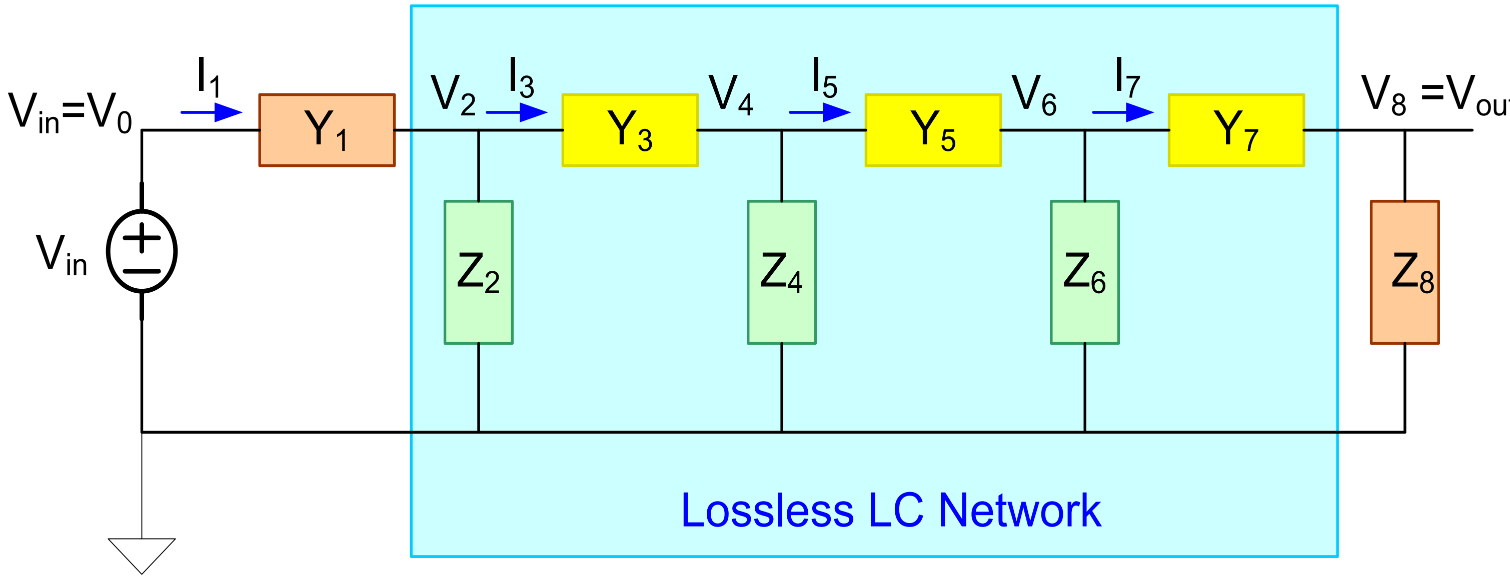




Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically in most integrated applications or even in pc-board based designs

Doubly-terminated Ladder Network with Low Passband Sensitivities



$$
\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}
$$

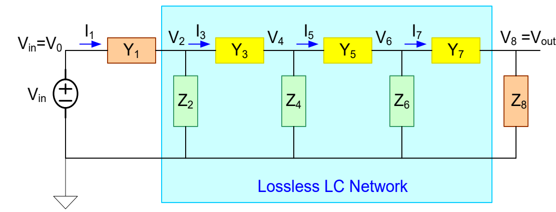
Complete set of independent equations that characterize this filter

Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Review from last lecture

Consider now only the set of equations and disassociate them from the circuit from where they came



$$V_1' = (V_0 - V_2)\frac{1}{R_1}$$

$$V_2 = (V_1' - V_3')\frac{1}{sC_2}$$

$$V_3' = (V_2 - V_4)\frac{1}{sL_3}$$

$$V_4 = (V_3' - V_5')\frac{1}{sC_4}$$

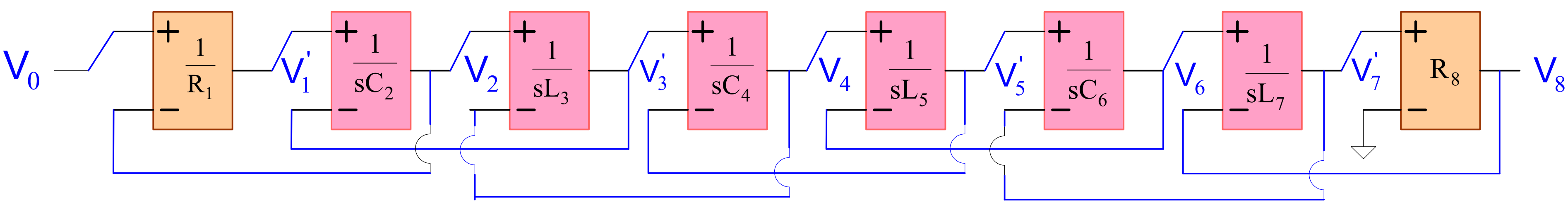
$$V_5' = (V_4 - V_6)\frac{1}{sL_5}$$

$$V_6 = (V_5' - V_7')\frac{1}{sC_6}$$

$$V_7' = (V_6 - V_8)\frac{1}{sL_7}$$

$$V_8 = V_7' R_8$$

The interconnections that complete each equation can now be added



Review from last lecture

Bandpass Leapfrog Structures

Consider lowpass to bandpass transformations

Un-normalized

$$s_n \rightarrow \frac{s^2+\omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2+\omega_0^2}$$

$$\frac{1}{s_n+\alpha} \rightarrow \frac{sBW}{s^2+s\alpha BW+\omega_0^2}$$

Normalized

$$s_n \rightarrow \frac{s^2+1}{sBW_n}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW_n}{s^2+1}$$

$$\frac{1}{s_n+\alpha} \rightarrow \frac{sBW_n}{s^2+s\alpha BW_n+1}$$

Review from last lecture

Bandpass Leapfrog Structures

Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2+\omega_0^2}$$

Integrators map to bandpass biquads with infinite Q

$$\frac{1}{s_n+\alpha} \rightarrow \frac{sBW}{s^2+s\alpha BW+\omega_0^2}$$

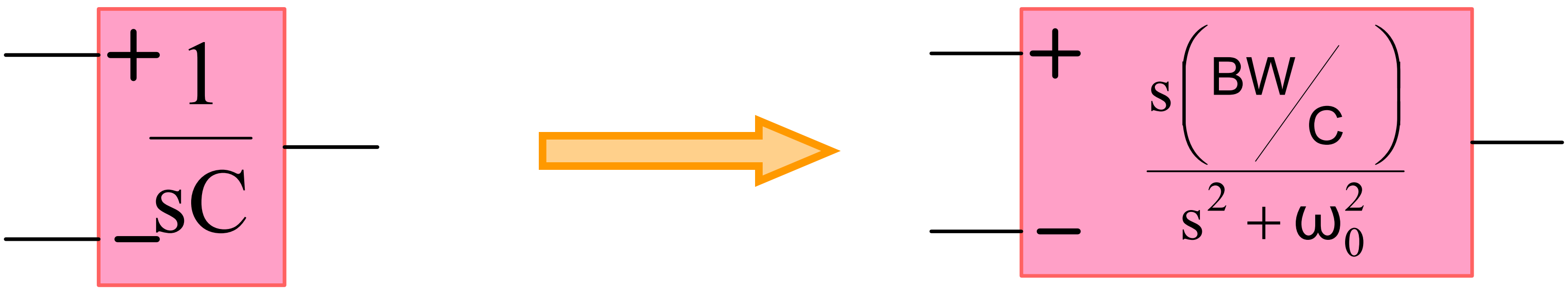
Lossy integrators map to bandpass biquads with finite Q

Invariably the resistance spread or the capacitance spread increases with Q

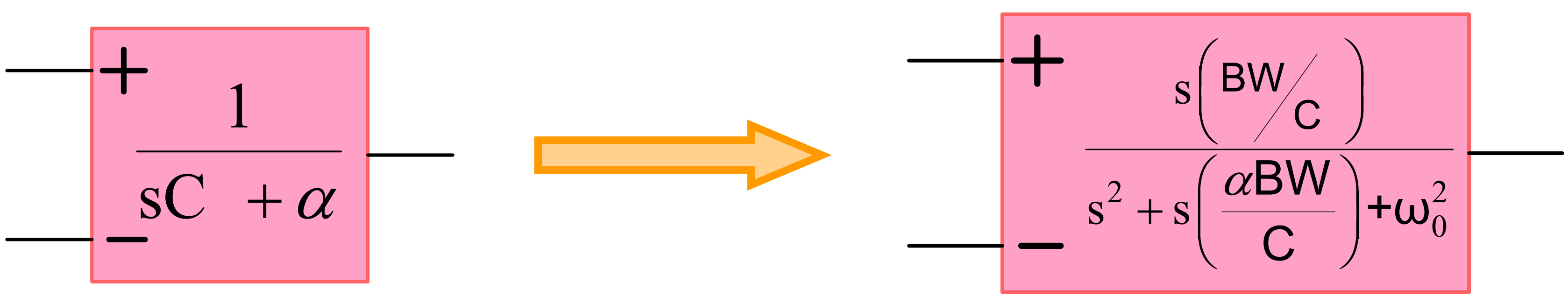
- Does this imply that the area will get very large if Q gets large?
- But what about infinite Q?
- Will infinite Q biquads be unstable?
- Is this a problem ?

Bandpass Leapfrog Structures

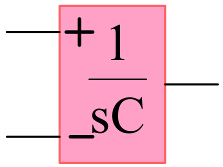
$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$



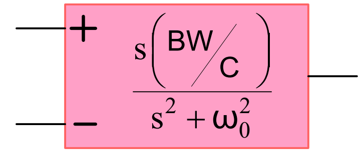
$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

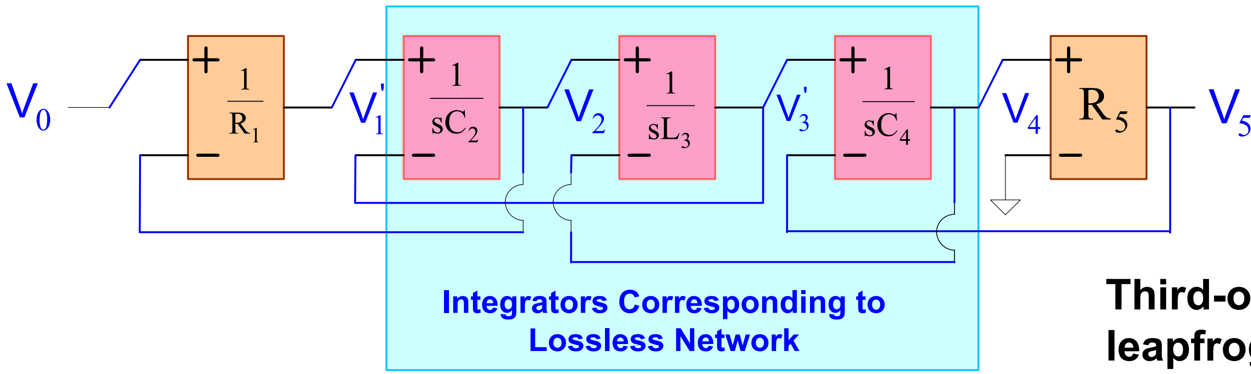


Bandpass Leapfrog Structures

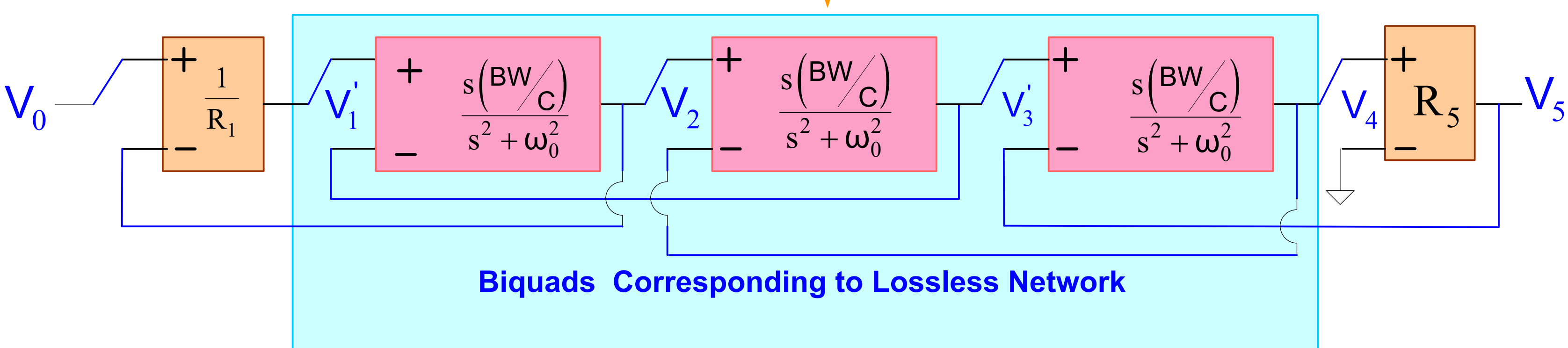


$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$



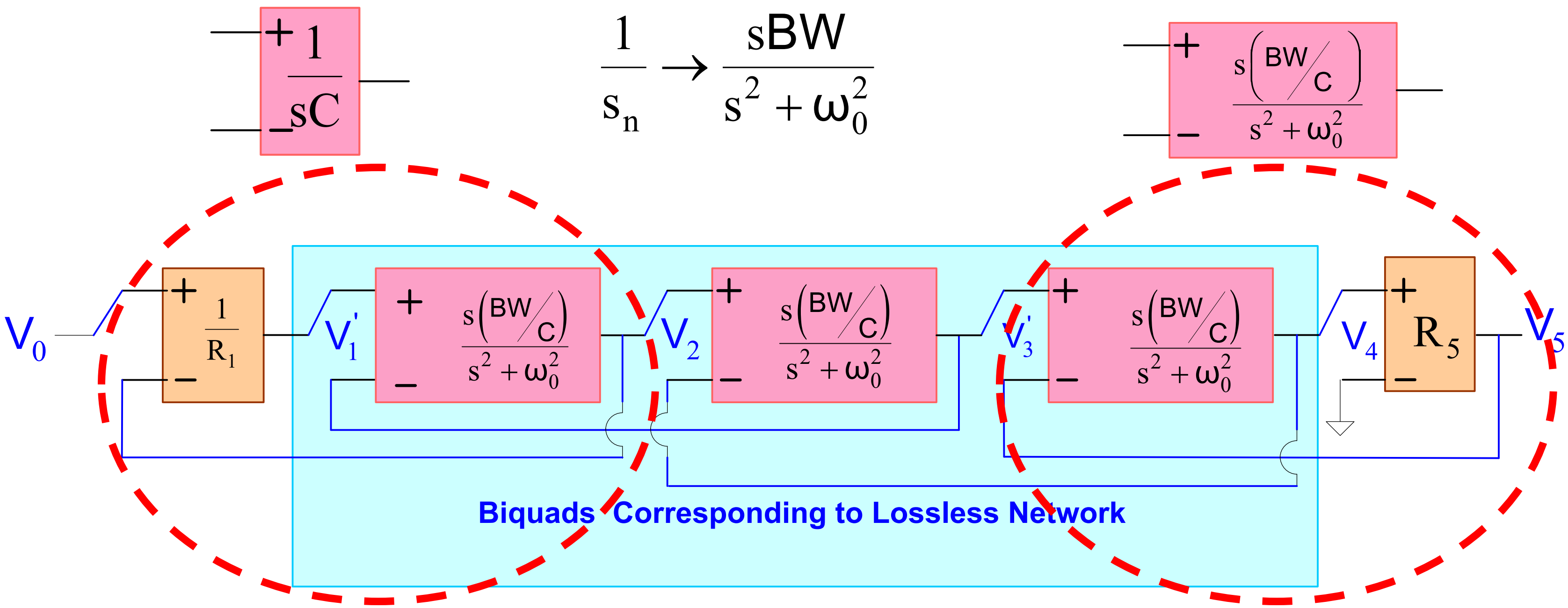


Third-order lowpass leapfrog filter



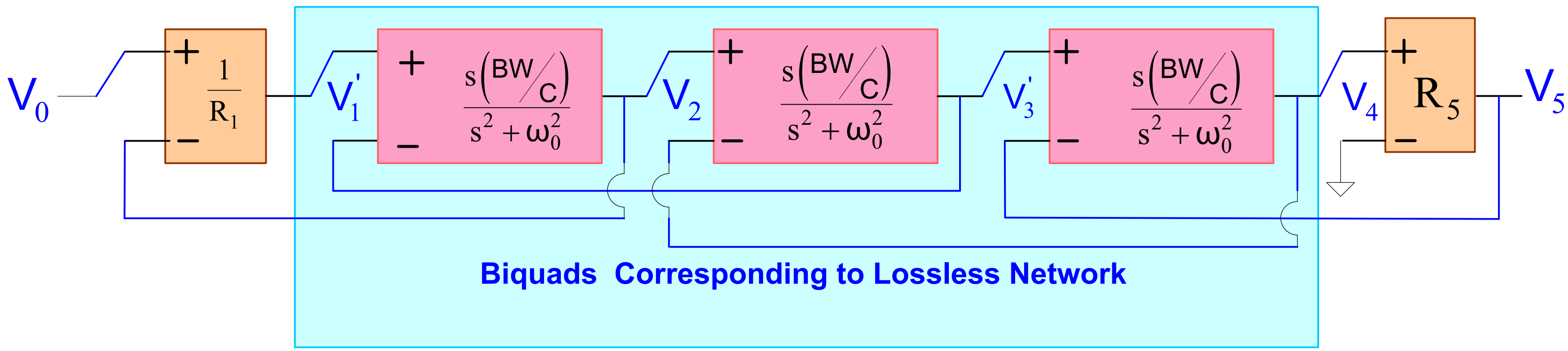
Sixth-order bandpass leapfrog filter

Bandpass Leapfrog Structures



"Loss" at input and/or output can usually be incorporated into finite-Q terminating biquads instead of requiring additional voltage amplifiers

Bandpass Leapfrog Structures



- The bandpass biquads can be implemented with various architectures and the architecture does not ideally affect the passband sensitivity of the filter

- Integrator-based biquads are often used in integrated applications

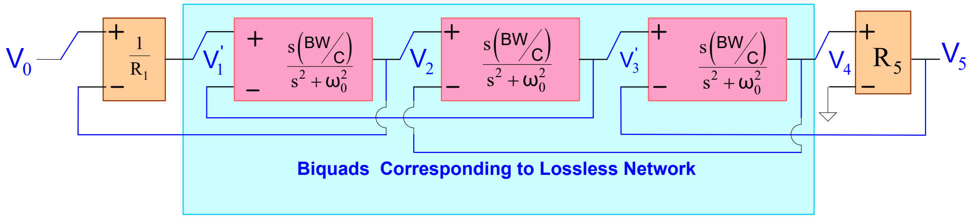
Note the lossless biquads are infinite Q structures !

It is easy and practical to implement infinite Q biquads

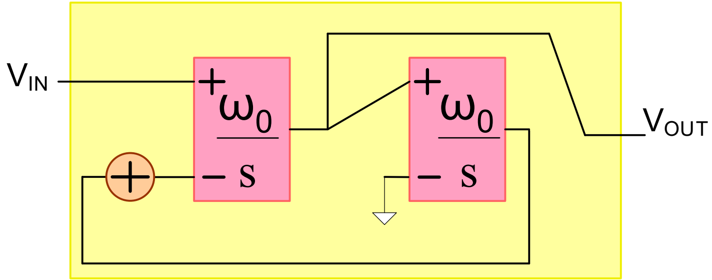
Stability of the infinite Q biquads is not of concern

Is it easy to trim a bandpass Leapfrog structure ?

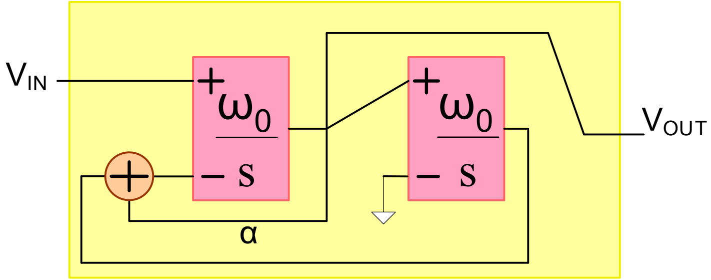
Bandpass Leapfrog Structures



Integrator-based biquads



$$T(s) = \frac{s\omega_0}{s^2 + \omega_0^2}$$



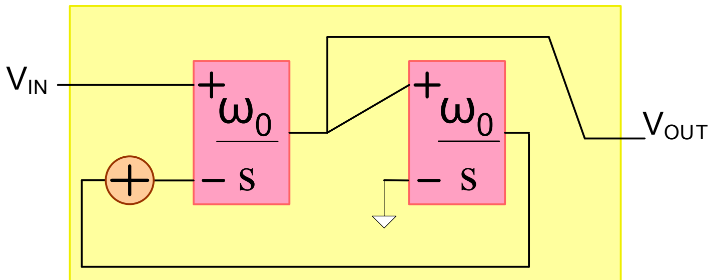
$$T(s) = \frac{s\omega_0}{s^2 + s\alpha\omega_0 + \omega_0^2}$$

Bandpass Leapfrog Structures

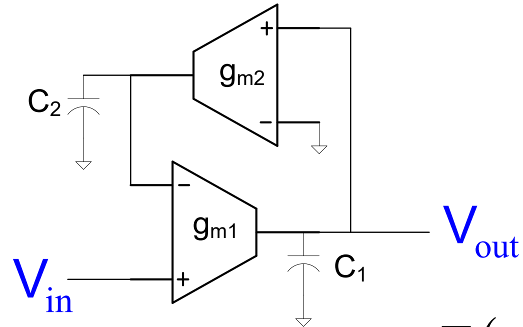
Integrator-based biquads

OTA-C Implementations (Concept)

Infinite Q bandpass biquad



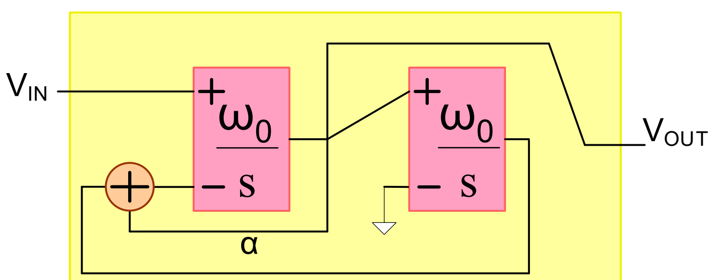
$$T(s)=\frac{s\omega_0}{s^2+\omega_0^2}$$



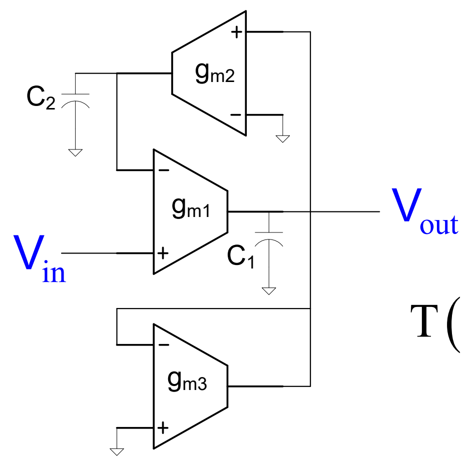
(Not Differential)

$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Finite Q bandpass biquad



$$T(s)=\frac{s\omega_0}{s^2+s\alpha\omega_0+\omega_0^2}$$



(Not Differential)

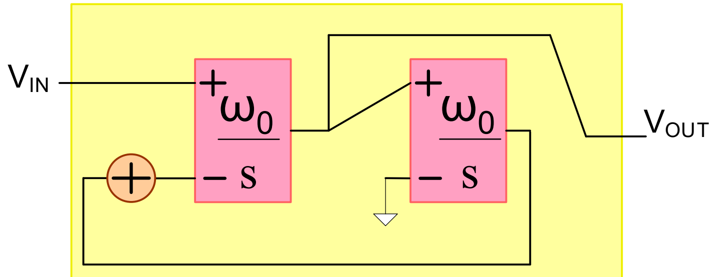
$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+s\frac{g_{m3}}{C_1}+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Bandpass Leapfrog Structures

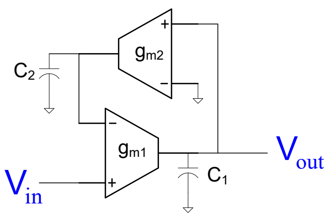
Integrator-based biquads

OTA-C Implementations

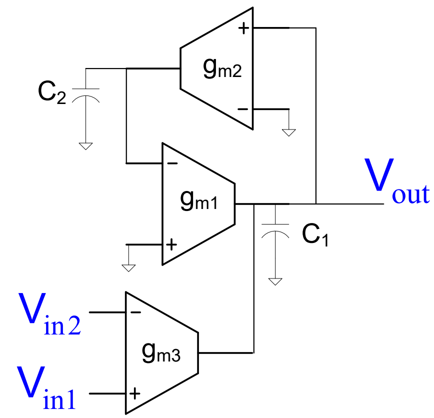
Infinite Q bandpass biquad



$$T(s) = \frac{s\omega_0}{s^2 + \omega_0^2}$$



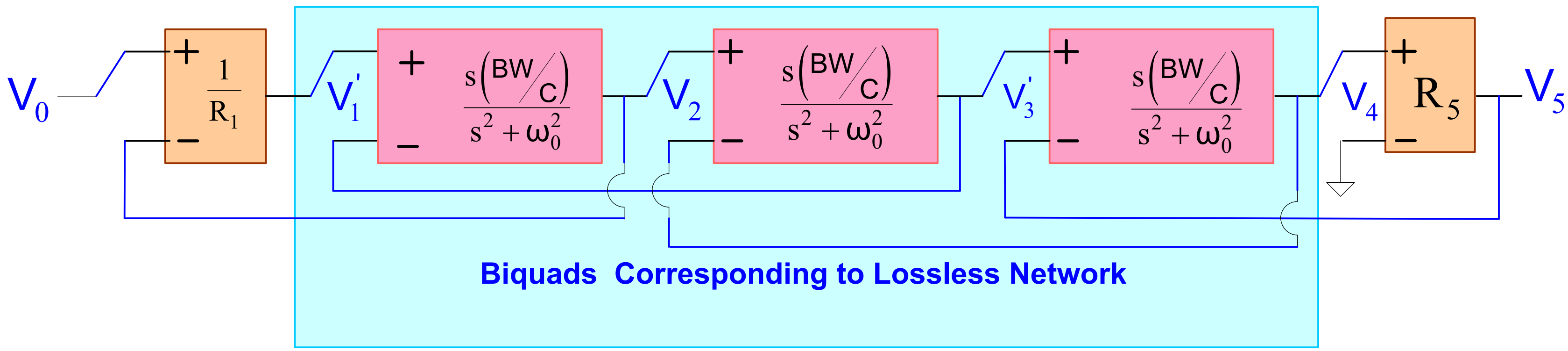
$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$



$$V_{OUT}(s) = \frac{s\left(\frac{g_{m3}}{C_1}\right)\left[V_{in1} - V_{in2}\right]}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Multiple inputs can be added to lossy integrator too!

Bandpass Leapfrog Structures



Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

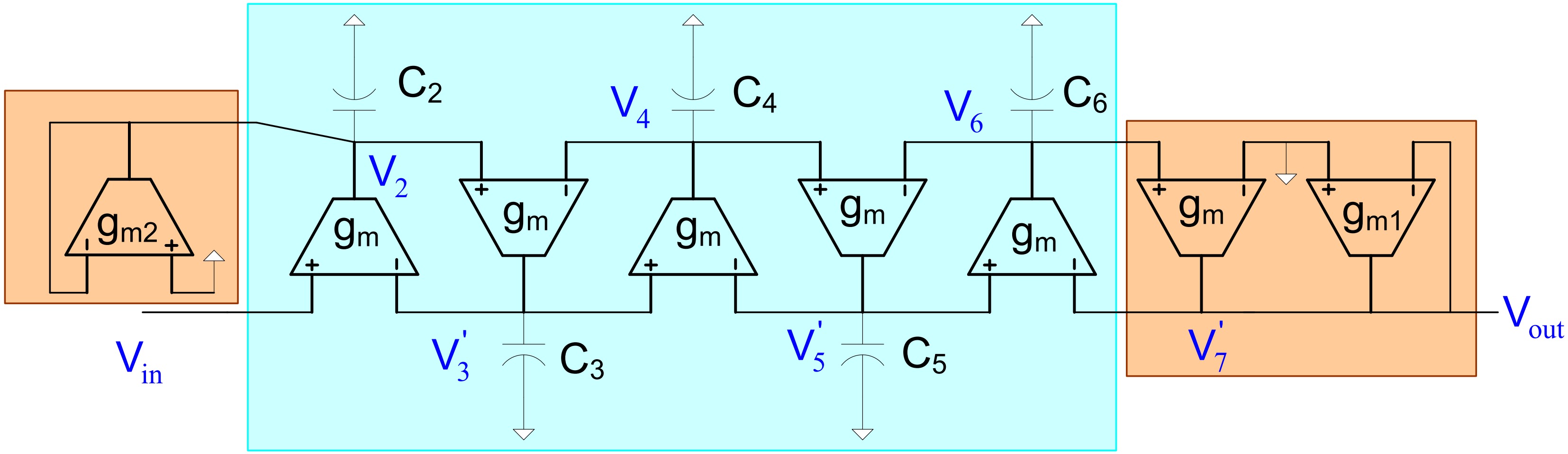
Yes – have shown by example in g_m-C family and also easy in other families

Are there stability concerns about the infinite Q biquads?

Stability of overall leapfrog structure of concern, not stability of individual biquads
Overall leapfrog structure is robust with low passband sensitivities !

Leapfrog Implementations

Fifth-order Lowpass Leapfrog with OTAs



$$V_1' = \frac{1}{R_1}\left(V_{in} - V_2\right)$$

$$V_2 = \frac{g_m / C_2}{s}\left(V_1' - V_3'\right)$$

$$V_3' = \frac{g_m / C_3}{s}\left(V_2 - V_4\right)$$

$$V_4 = \frac{g_m / C_4}{s}\left(V_3' - V_5'\right)$$

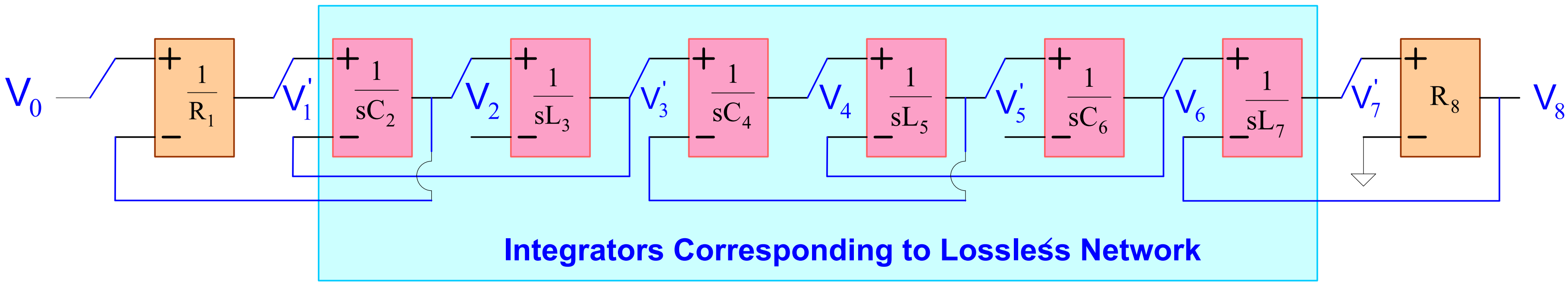
$$V_5' = \frac{g_m / C_5}{s}\left(V_4 - V_6\right)$$

$$V_6 = \frac{g_m / C_6}{s}\left(V_5 - V_7'\right)$$

$$V_7' = \left(g_m / g_{m1}\right) V_6$$

Practically can either fix g_ms and vary capacitors or fix capacitors and vary g_m's

Some leapfrog properties



What can be said about sensitivities of parameters such as band edges of leapfrog filters? If these sensitivities are not at or near 0, are they at least very small?

No! Nothing can be said about these sensitivities and they are not necessarily any smaller than what one may have for other structures such as cascaded biquads

Instead of having components (such as L's or C's) in the image of the lossless ladder network there are circuits such as integrators or biquads with more than one characterization parameters. Are the sensitivities of $|T(j\omega)|$ to these components also 0 at frequencies where the "parent" passive filter are 0?

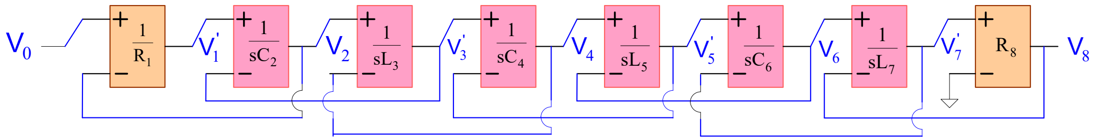
Yes! The following theorem addresses this issue in the case of integrators

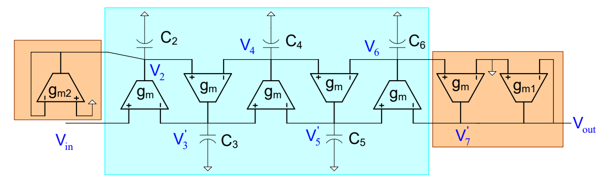
Theorem: If f(u) is a function of a variable u where $u=x_1x_2$, then

$$S_u^f = S_{x_1}^f + S_{x_2}^f$$

It can be shown that if the unity gain frequency of an integrator which may be expressed (for example) as 1/RC, then the transfer function magnitude sensitivity to both R and C vanish at frequencies where the sensitivity to I_0 vanishes

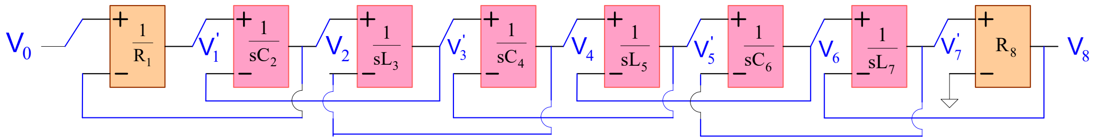
Leapfrog Filters
A Seminal Contribution

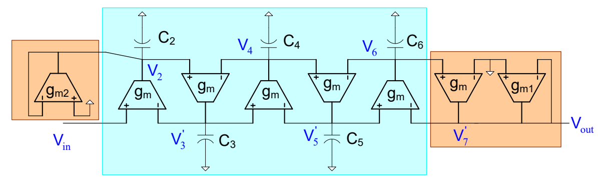




- A valuable contribution ?
- A timely contribution ?
- A clever idea?
- Would someone else have come up with it had Girling and Good not made the discovery?
- Example of unlikely publication making major disclosure

Leapfrog Filters
A Seminal Contribution





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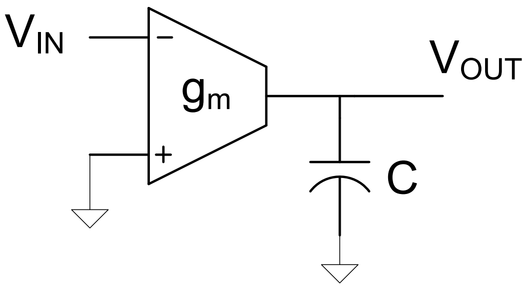
Transconductor Design



Transconductor-based filters depend directly on the g_m of the transconductor

Feedback is not used to make the filter performance insensitive to the transconductance gain

Linearity and spectral performance of the filter strongly dependent upon the linearity of the transconductor

Often can not justify elegant linearization strategies in the transconductors because of speed, area, and power penalties

Seminal Work on the OTA

RCA

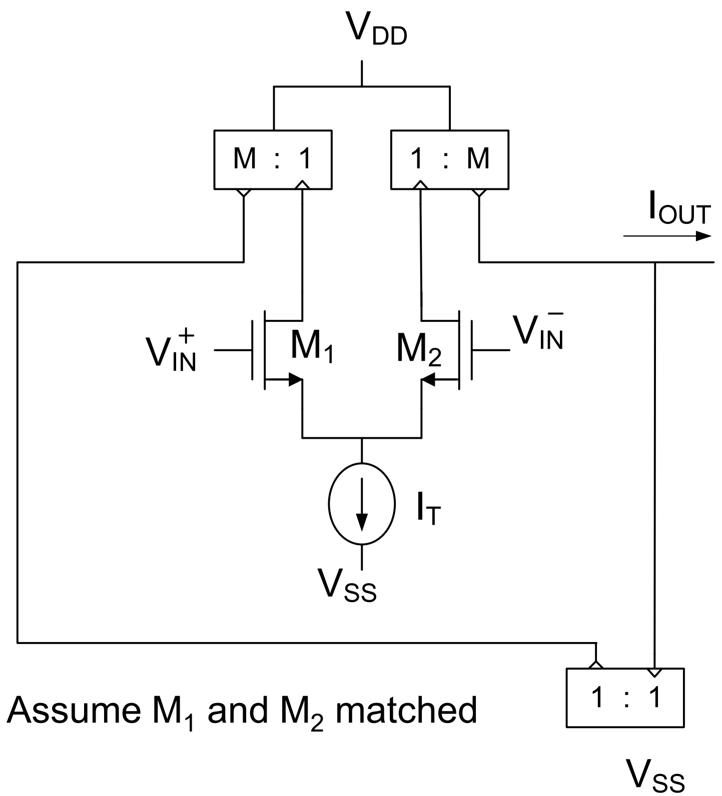
OTA Obsoletes Op Amp

by C.F. Wheatley
H.A. Wittlinger

From:

1969 N.E.C. PROCEEDINGS
December 1969

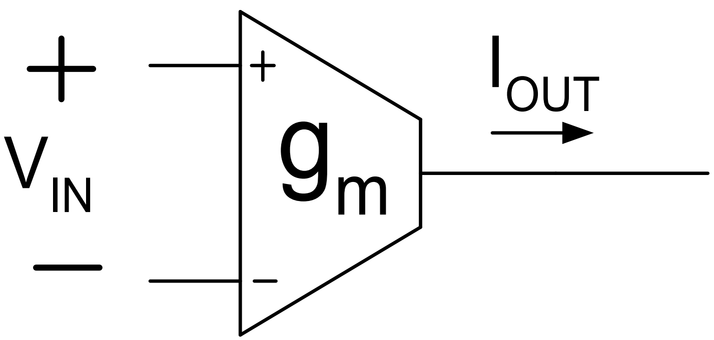
Current Mirror Op Amp W/O CMFB



Assume M_1 and M_2 matched

$$g_{mEQ} = Mg_{m1}$$

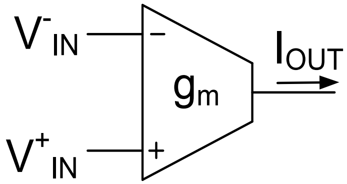
Often termed an OTA

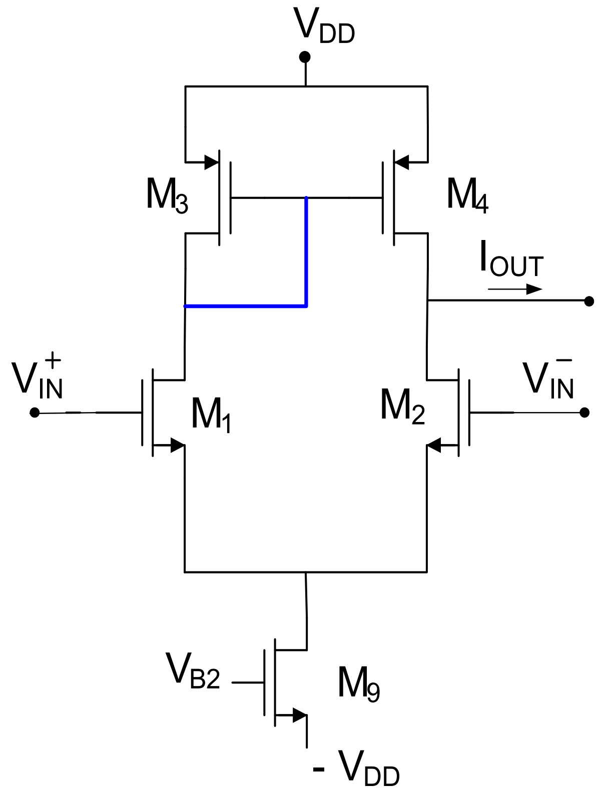


Introduced by Wheatley and Whitlinger in 1969

$$I_{OUT} = g_m V_{IN}$$

Basic OTA based upon differential pair

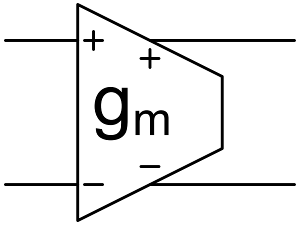


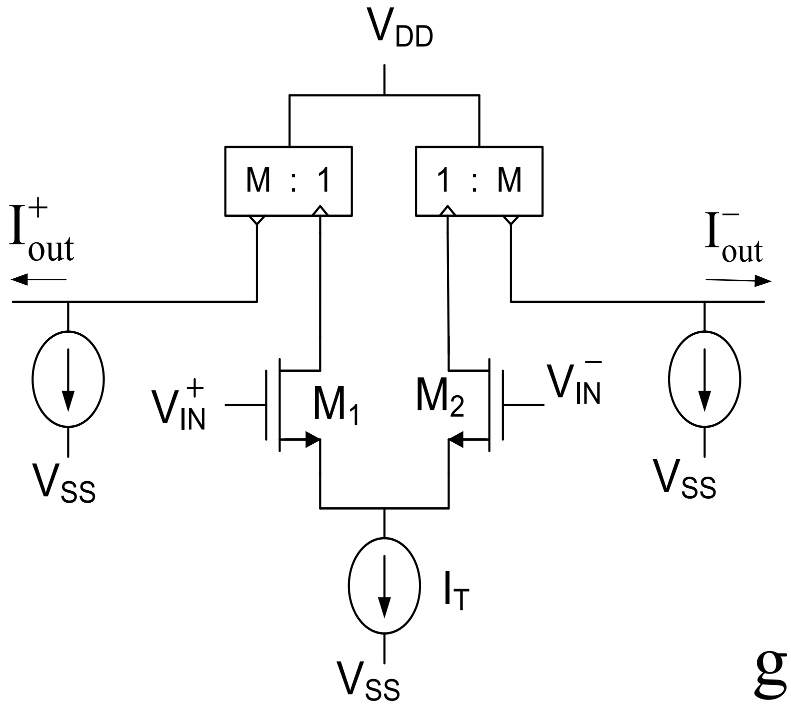


$$g_m = g_{m1}$$

Assume M_1 and M_2 matched,
M_3 and M_4 matched

Differential output OTA based upon differential pair

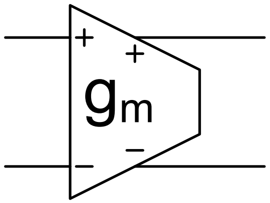


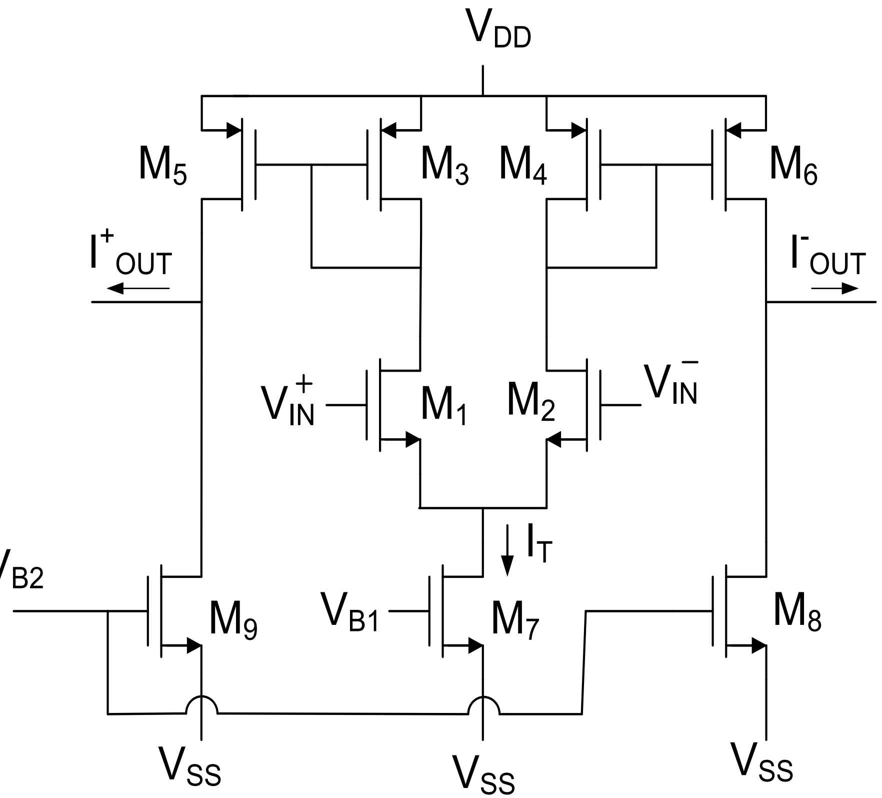


$$g_m = \frac{g_{m1}}{2} M$$

CMFB needed for the two output biasing current sources

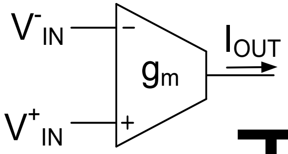
Differential output OTA based upon differential pair



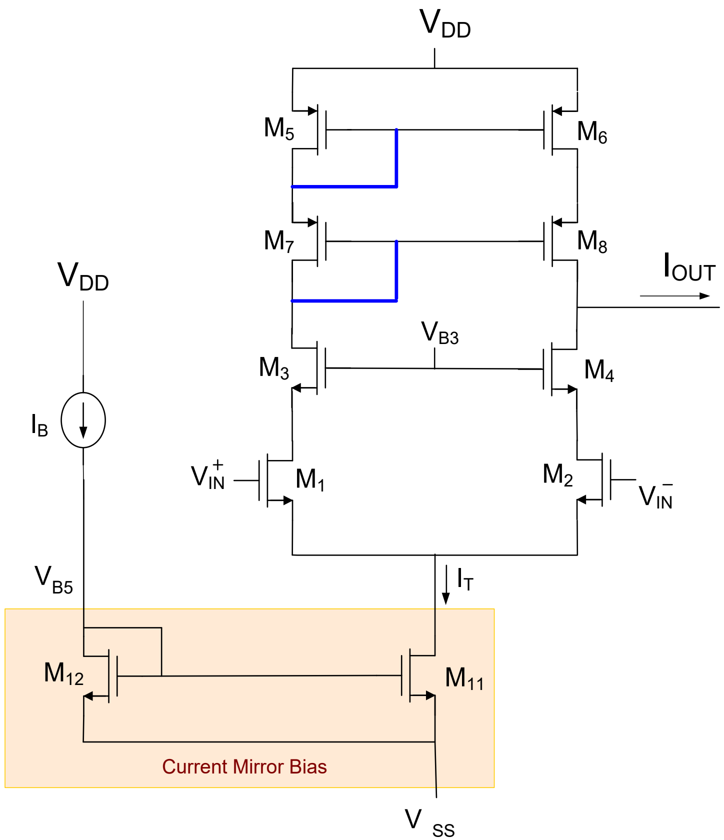


$$g_m = \frac{g_{m1}}{2} M$$

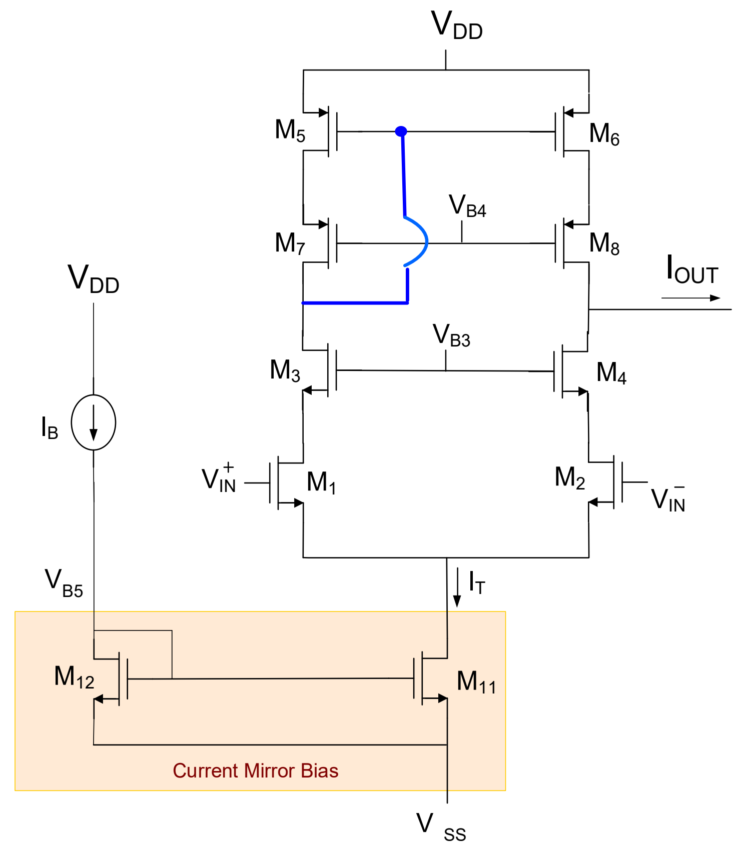
CMFB needed for the two output biasing current sources



Telescopic Cascode OTA



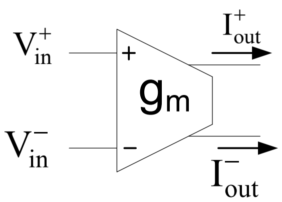
Standard p-channel Cascode Mirror

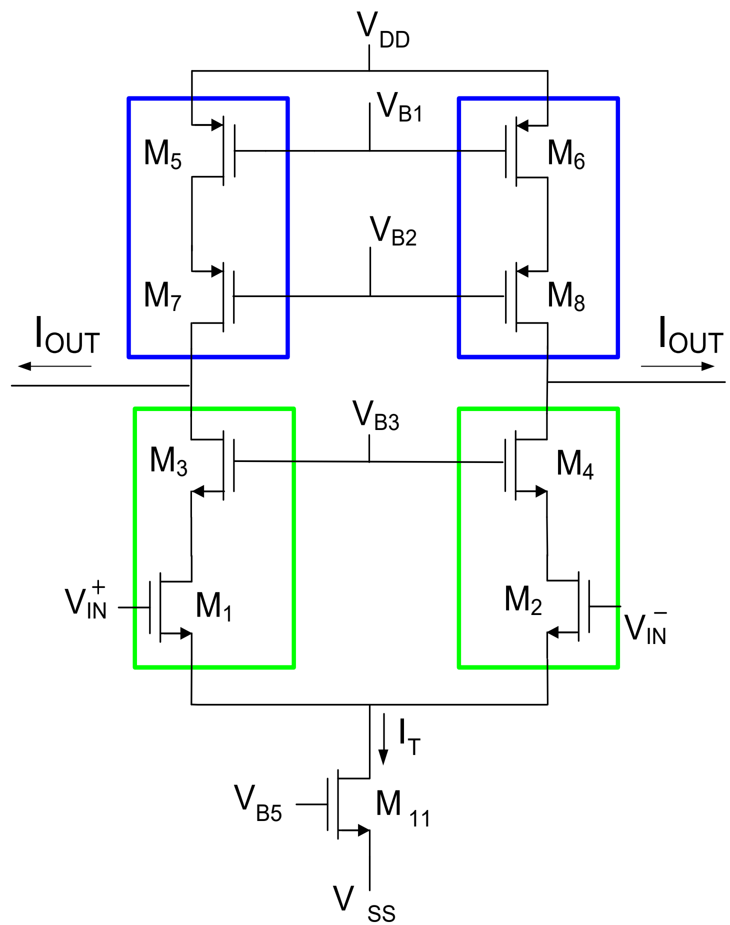


Wide-Swing p-channel Cascode Mirror

- Current-Mirror p-channel Bias to Eliminate CMFB
- Only single-ended output available

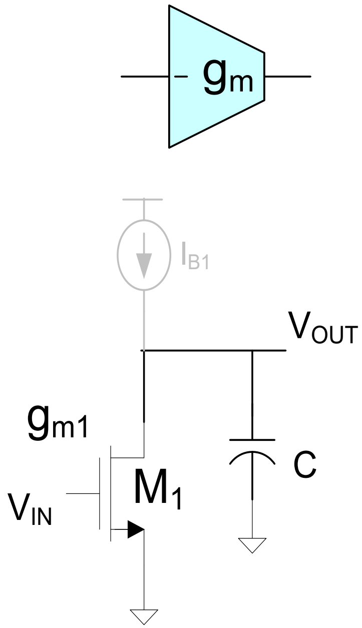
Telescopic Cascode OTA

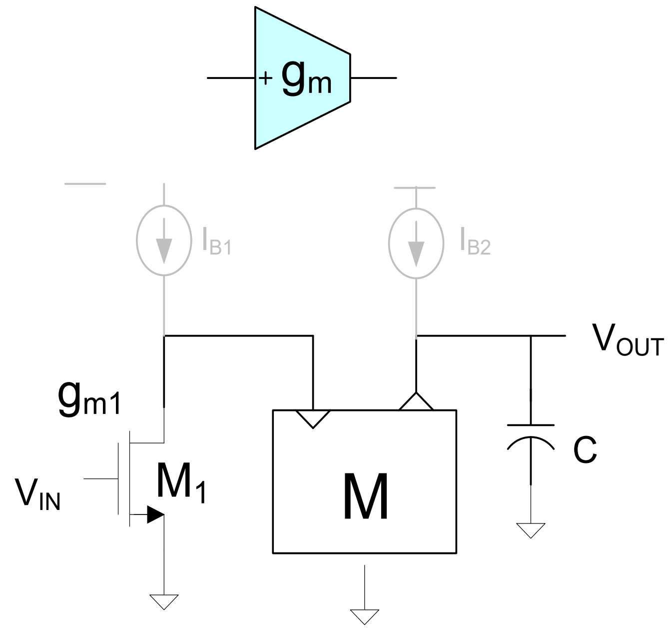




CMFB needed

Single-ended High-Frequency TA

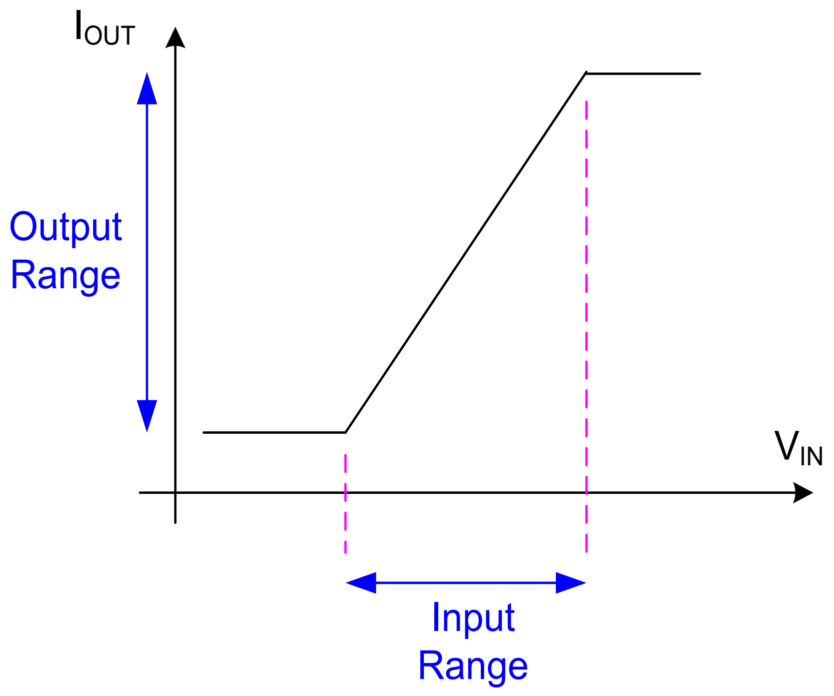




$$g_m = -g_{m1}$$

$$g_m = M g_{m1}$$

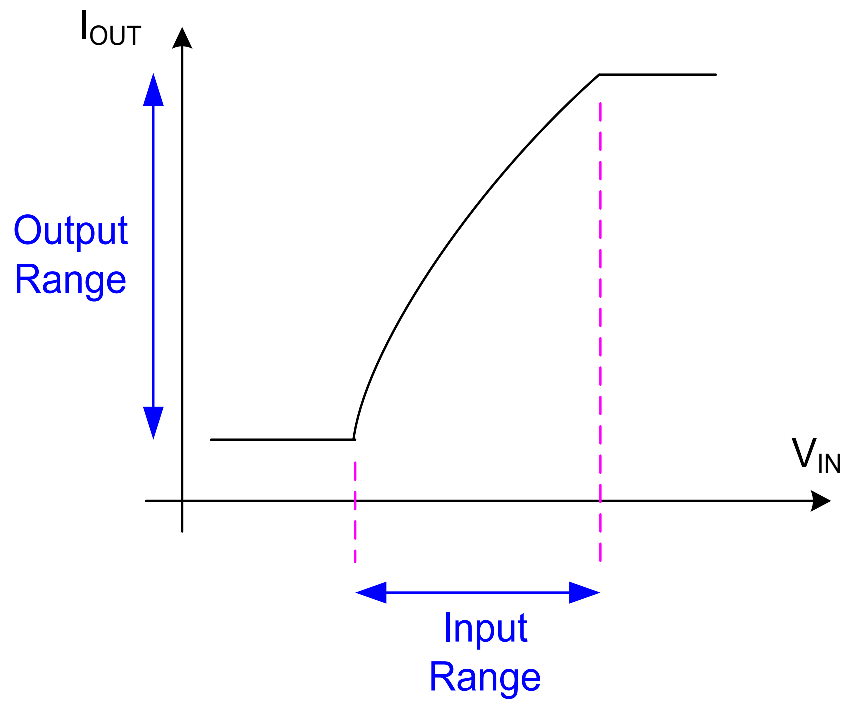
Signal Swing and Linearity



Ideal Scenario:

Completely Linear over Input and Output Range

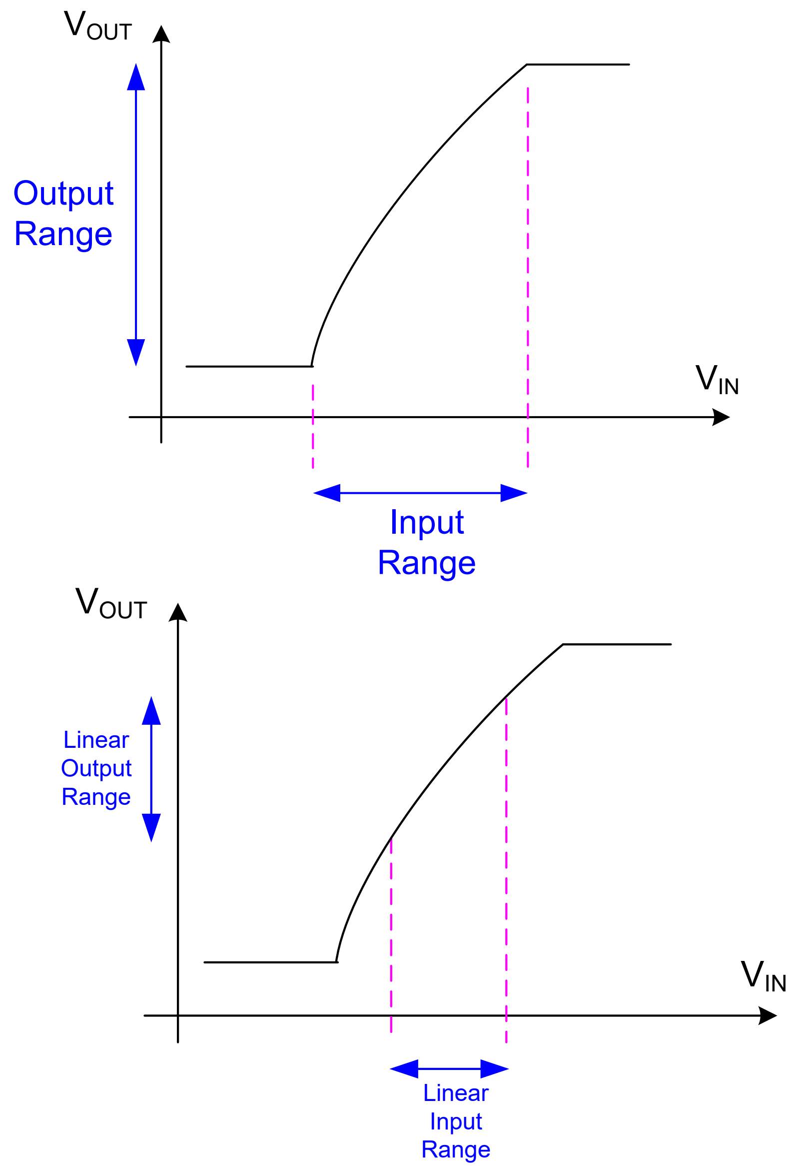
Signal Swing and Linearity



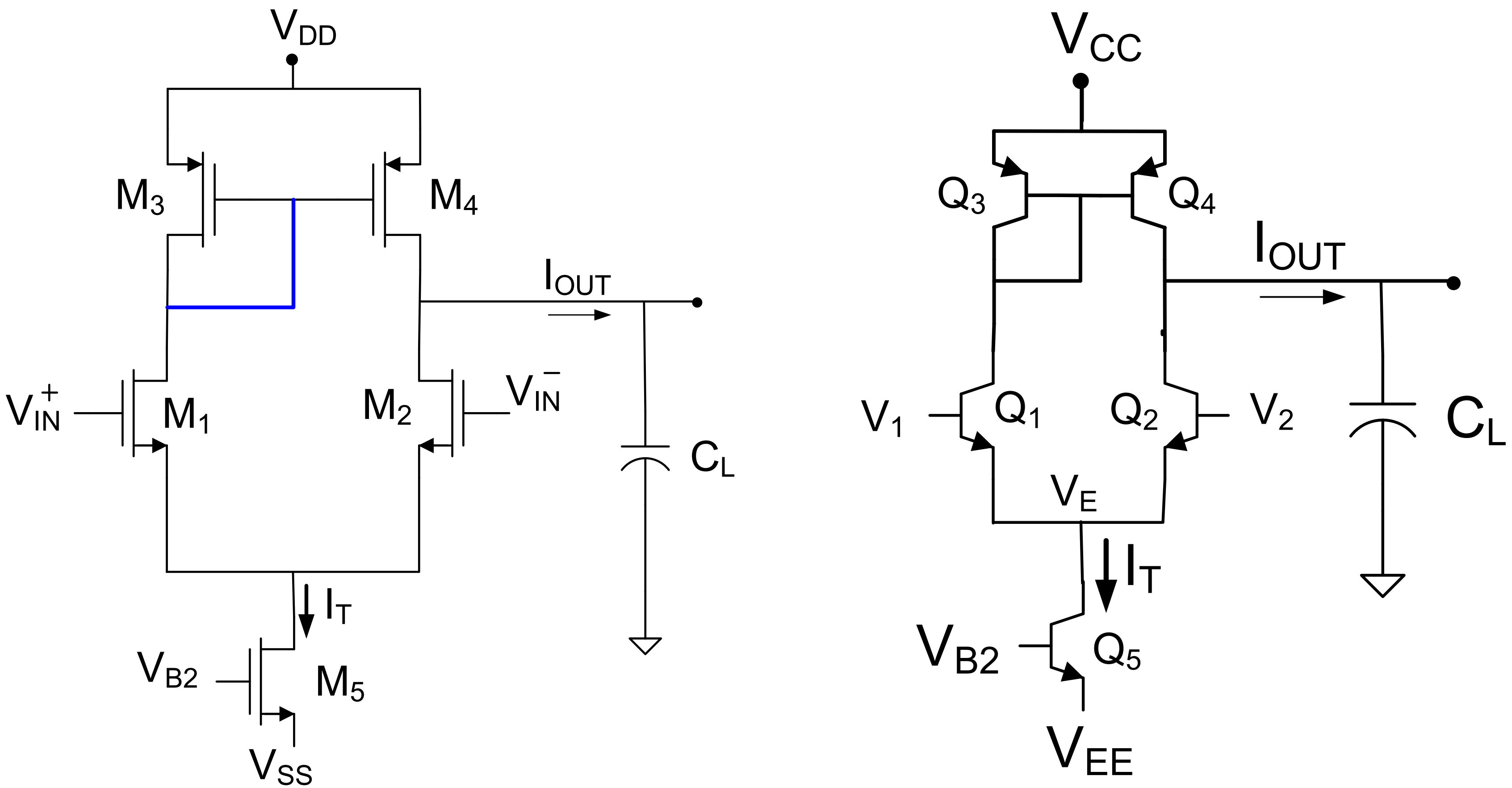
Realistic Scenario:

- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range

Signal Swing and Linearity

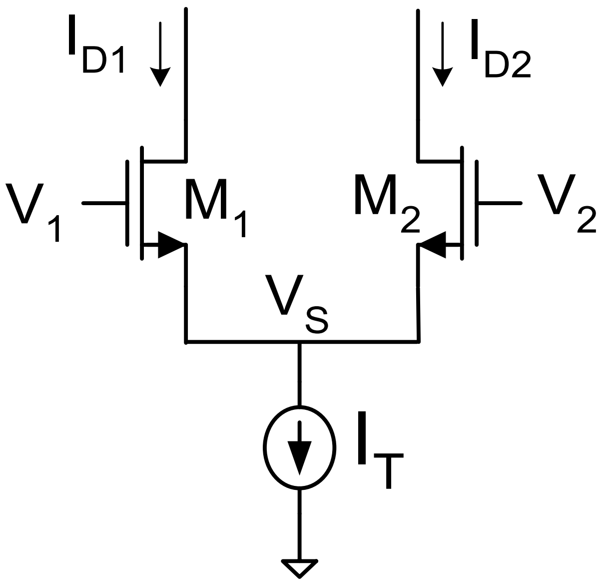


Linearity of Amplifiers

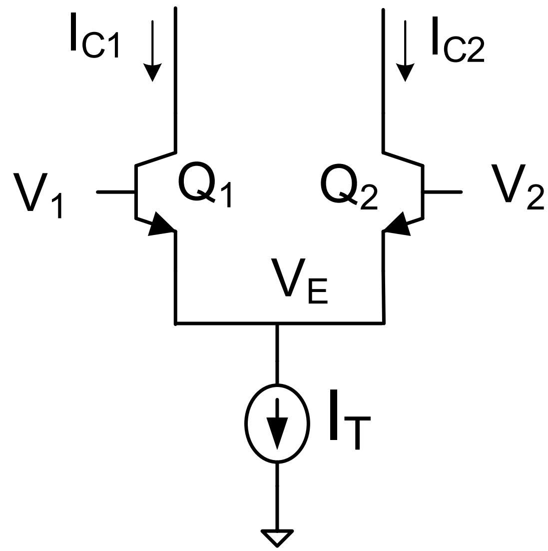


Strongly dependent upon linearity of transconductance of differential pair

Differential Input Pairs

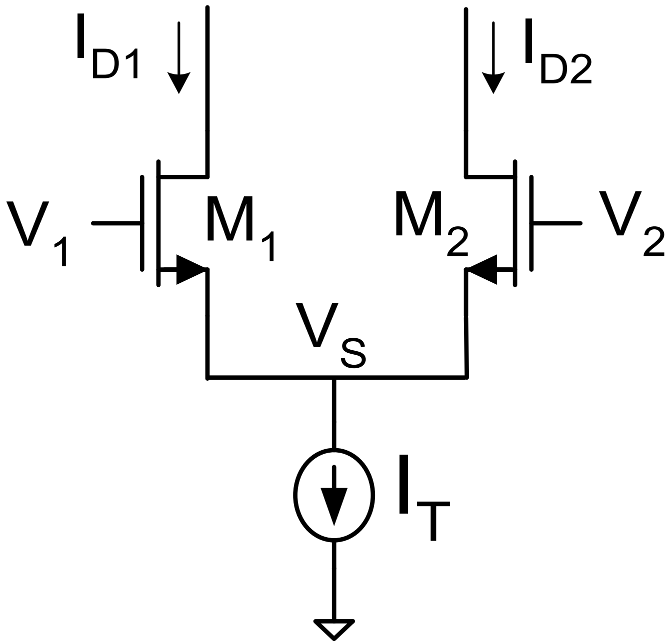


MOS Differential Pair



Bipolar Differential Pair

MOS Differential Pair



$$I_{D1} = \frac{\mu C_{OX} W}{2L}(V_1 - V_S - V_T)^2$$

$$I_{D2} = \frac{\mu C_{OX} W}{2L}(V_2 - V_S - V_T)^2$$

$$I_{D1} + I_{D2} = I_T$$

$$\sqrt{I_{D1}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_1 - V_S - V_T$$

$$\sqrt{I_{D2}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_2 - V_S - V_T$$

$$V_d = V_2 - V_1$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

MOS Differential Pair

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

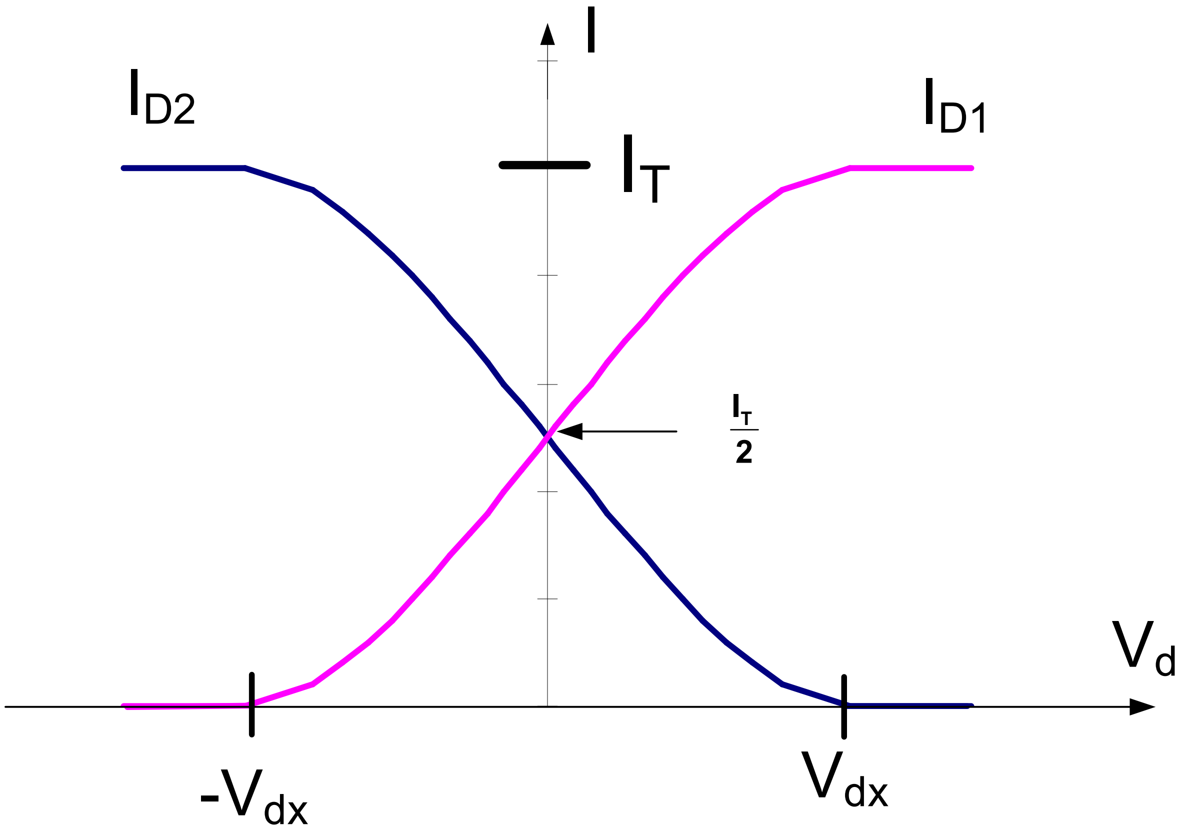
$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

What values of V_d will cause all of the current to be steered to the left or the right ?

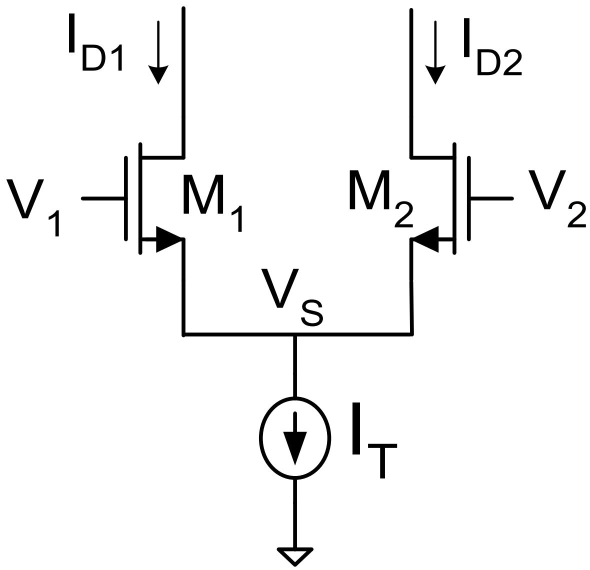
$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

Transfer Characteristics of MOS Differential Pair

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right) \qquad V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$



Q-point Calculations for MOS Differential Pair



$$\frac{I_T}{2} = \frac{\mu C_{OX} W}{2L} \left(V_{EB}\right)^2$$

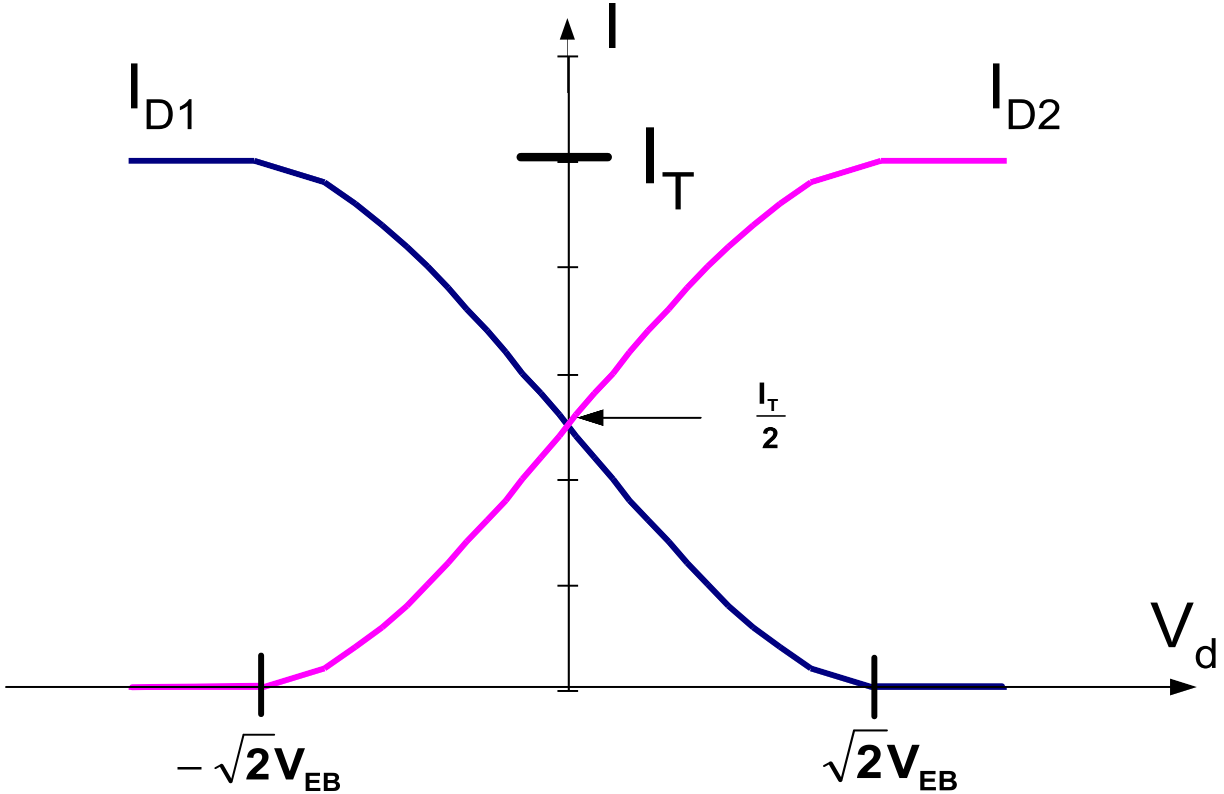
$$V_{EB} = \sqrt{I_T}\sqrt{\frac{L}{\mu C_{OX} W}}$$

Observe !!

$$V_{dx} = \pm\sqrt{2}V_{EB}$$

Transfer Characteristics of MOS Differential Pair

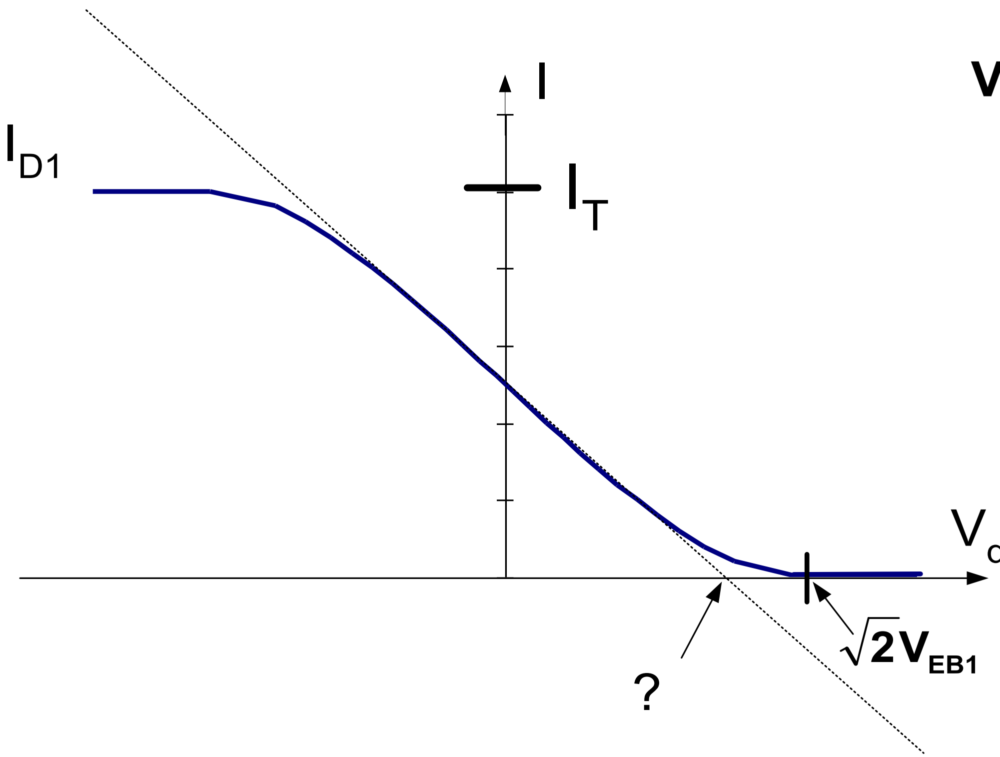
$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$



V_{EB} affects linearity

How linear is the amplifier ?

How linear is the amplifier ?



$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

Consider the fit line:

$$I = mV_d + h$$

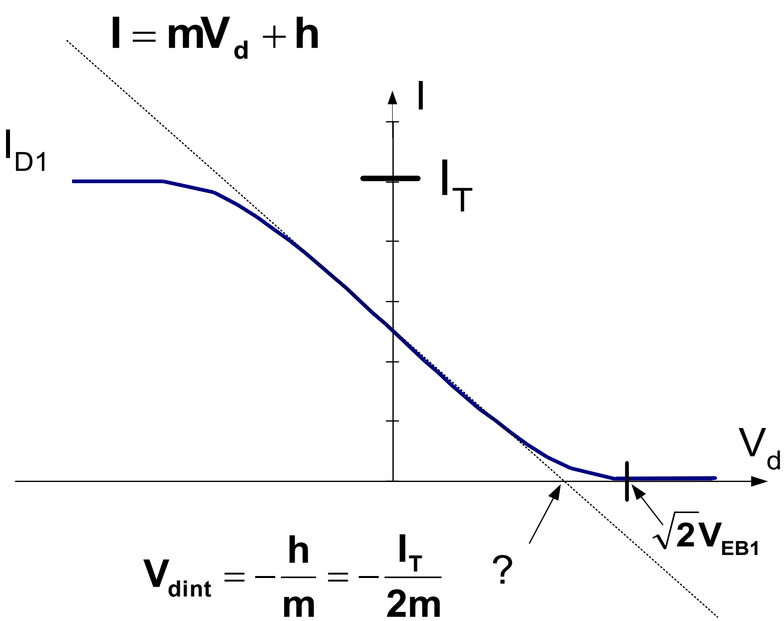
When $V_d=0$, $I=I_T/2$, thus

$$h = \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

How linear is the amplifier ?



$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$\frac{\partial V_d}{\partial I_{D1}} = \sqrt{\frac{2L}{\mu C_{OX} W}}\left.\left(\frac{1}{2}(I_T - I_{D1})^{-1/2}(-1) - \frac{1}{2}(I_{D1})^{-1/2}\right)\right|_{Q-point}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\sqrt{\frac{L}{\mu C_{OX} W}}\sqrt{\frac{1}{I_T}}$$

$$\sqrt{\frac{L}{\mu C_{OX} W}} = \frac{V_{EB1}}{\sqrt{I_T}}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\frac{V_{EB1}}{I_T}$$

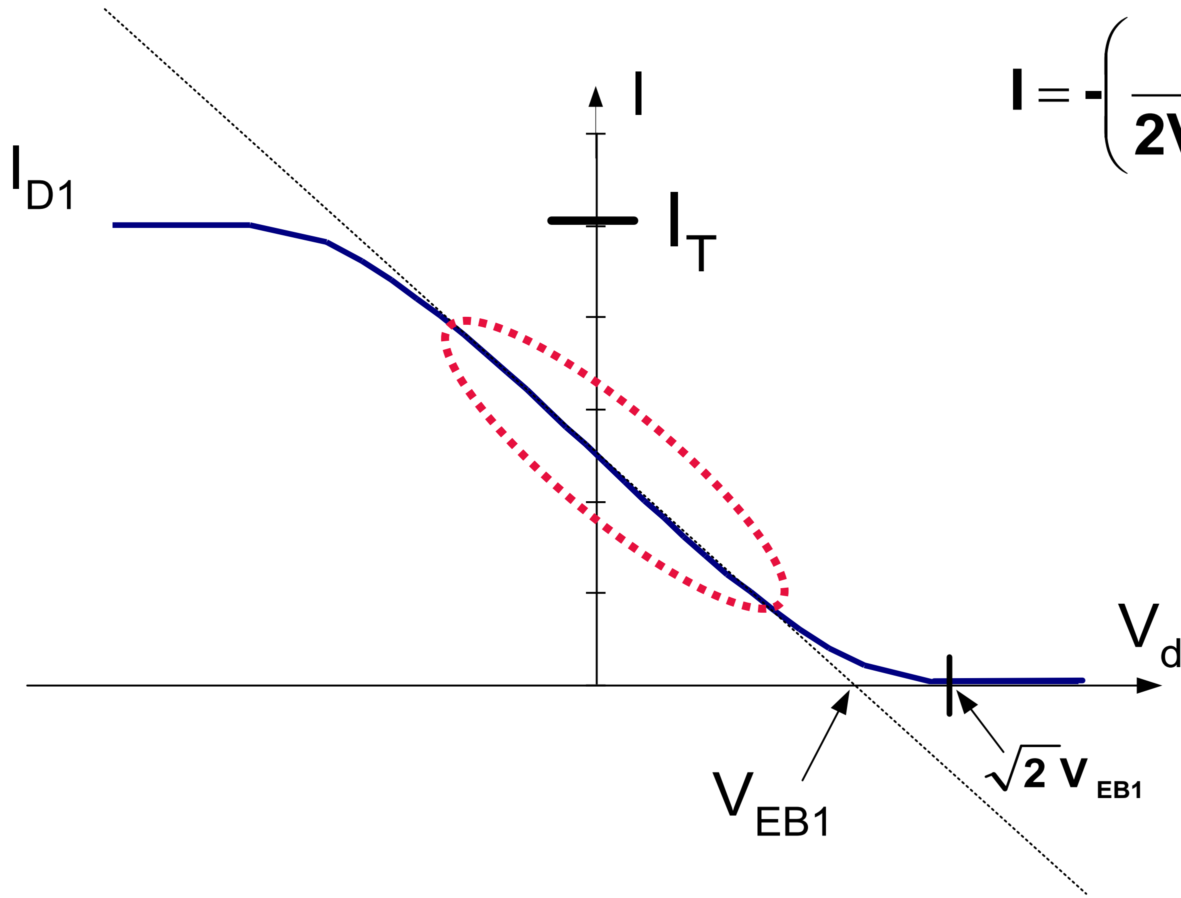
$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt} = -\frac{I_T}{2V_{EB1}}$$

$$V_{dint} = V_{EB1}$$

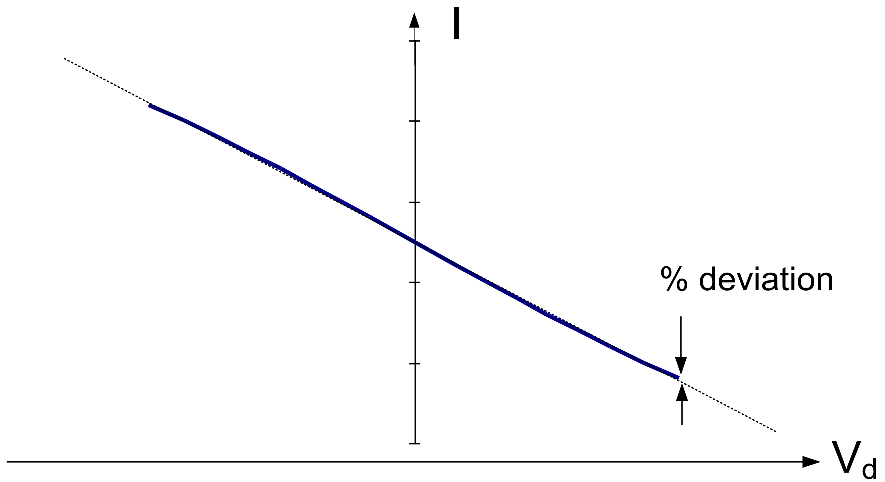
How linear is the amplifier ?

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?

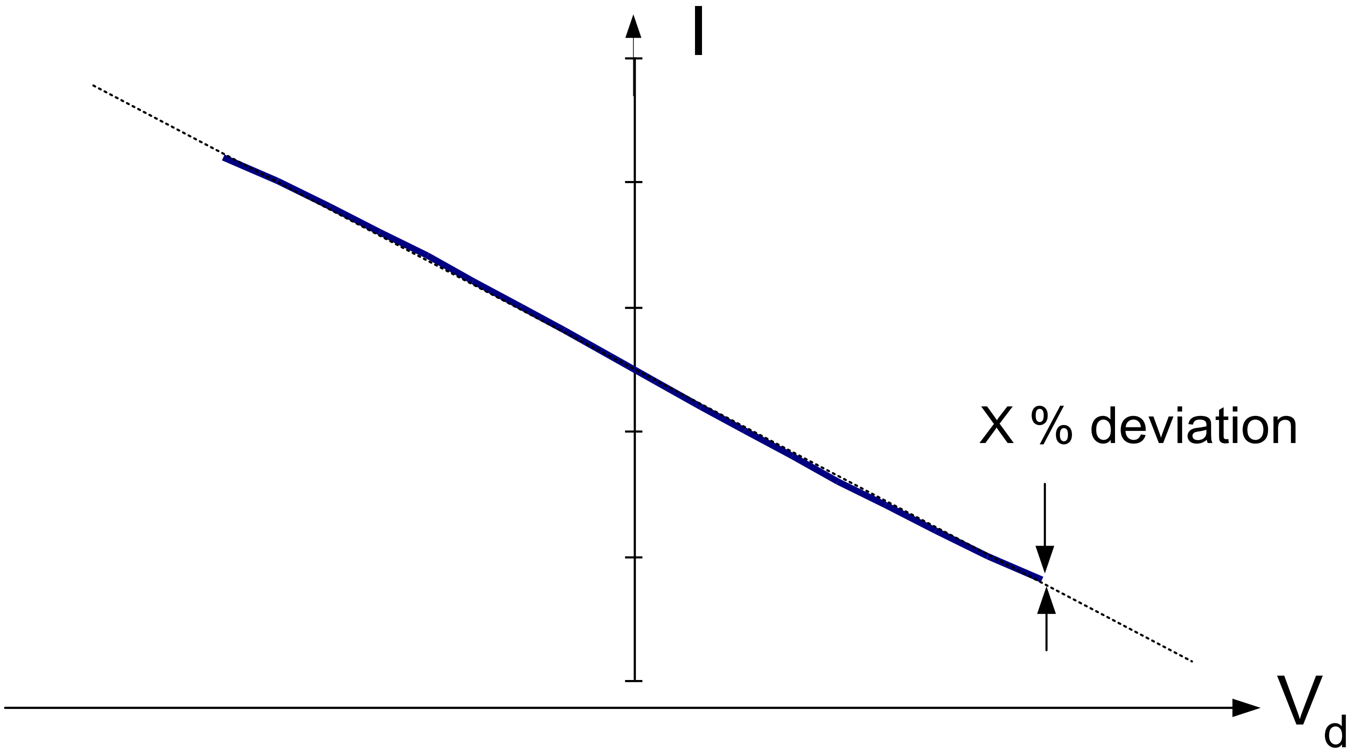


It can be shown that the deviation from the line in % is given by

$$\theta = 100\% \left(1 - \sqrt{1 - \frac{\left(V_d / V_{EB} \right)^2}{4}} \right)$$

Vd/VEB	θ	Vd/VEB	θ	Vd/VEB	θ
0.02	0.005	0.22	0.607	0.42	2.23
0.04	0.020	0.24	0.723	0.44	2.45
0.06	0.045	0.26	0.849	0.46	2.68
0.08	0.080	0.28	0.985	0.48	2.92
0.1	0.125	0.3	1.13	0.5	3.18
0.12	0.180	0.32	1.29	0.52	3.44
0.14	0.245	0.34	1.46	0.54	3.71
0.16	0.321	0.36	1.63	0.56	4.00
0.18	0.406	0.38	1.82	0.58	4.30
0.2	0.501	0.4	2.02	0.6	4.61

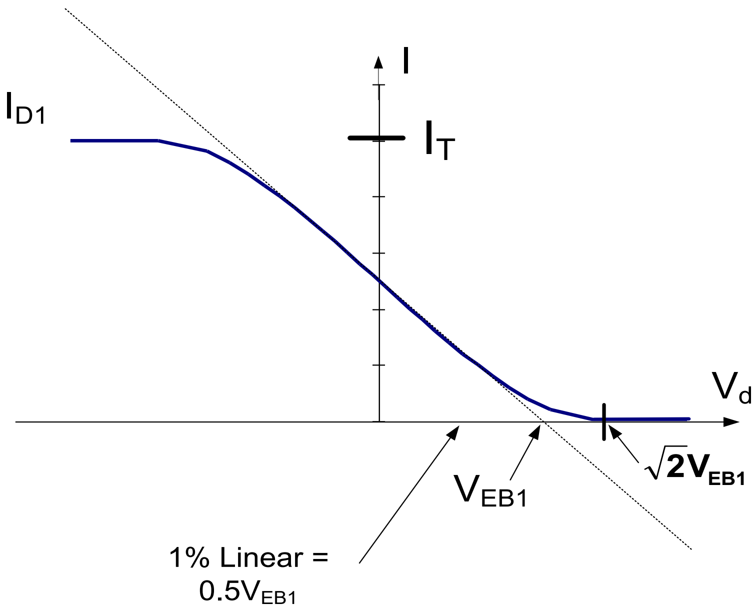
How linear is the amplifier ?



A 1% deviation from the straight line occurs at

$V_d \cong 0.3 V_{EB}$ and a 0.1% variation occurs at $V_d \cong \frac{V_{EB}}{10}$

What swings on drain currents are typical when using the differential pair in a voltage amplifier (OP AMP)?



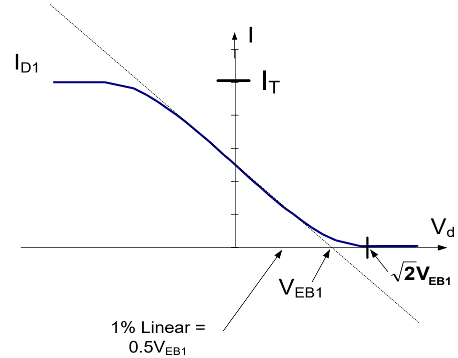
Assume the differential amplifier is the input stage to an op amp with gain Av and signal swing V_{OUTpp}

The differential swing at the input is thus

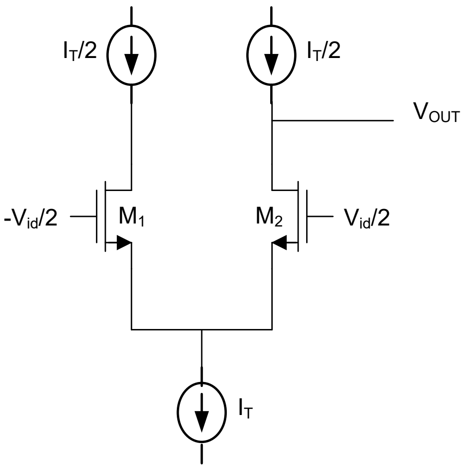
$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$

What swings on drain currents are typical when using the differential pair in a voltage amplifier (Op Amp)?

$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$



If the amplifier is the simple differential amplifier with current source loads



$$A_V = -\frac{g_{m1}}{2g_0} = \frac{2I_{DQ}/V_{EB1}}{2\lambda I_{DQ}}$$

$$A_V = -\frac{1}{\lambda V_{EB1}}$$

$$V_{INpp} = (\lambda V_{OUTpp})V_{EB1}$$

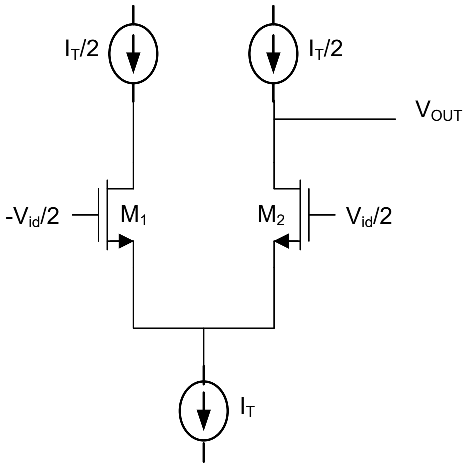
If $\lambda = .01V^{-1}$ and $V_{OUTpp} = 5V$,

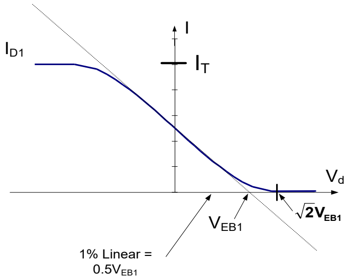
$$V_{INpp} = 0.05V_{EB1}$$

- This results in a very small nonlinearity in the Op Amp even with very large swings on the output.
- The current change is also very small
- When used in two-stage voltage amplifier structure, the nonlinearity in this structure is even much smaller!

What swings on drain currents are typical when using the differential pair in a voltage amplifier (Op Amp)?

$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$





If $\lambda = .01V^{-1}$

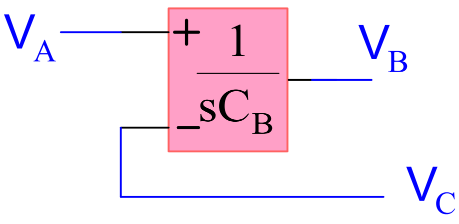
$$V_{INpp} = 0.05V_{EB1}$$

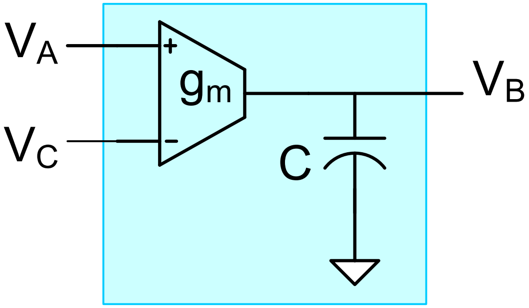
- This results in a very small nonlinearity in the Op Amp even with very large swings on the output.
- The current change is also very small
- When used in two-stage voltage amplifier structure, the nonlinearity in this structure is even much smaller!

Does this imply that large swings on the output introduce very little nonlinearity when used as an OTA?

No ! Because when used as an OTA the voltage swings in the input and output are often about the same!

Programmable Filter Structures





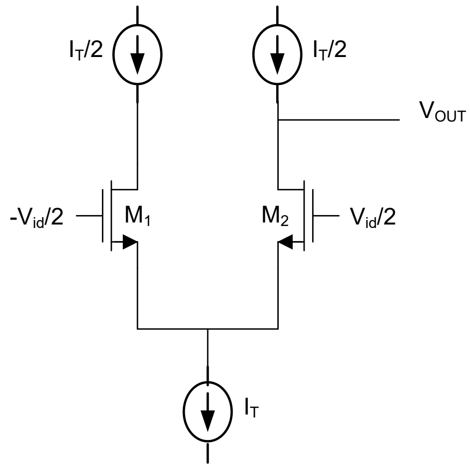
$$|\omega_0| = \frac{g_m}{C}$$

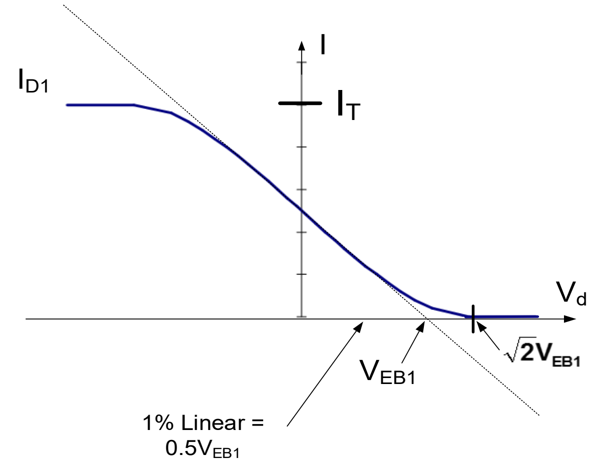
Often want to program or trim filters (i.e. trim ω_0)

Applicable in wide variety of filter architectures (here showing integrator-based)

Attractive to do this by adjusting g_m , in part, because g_m can be continuously adjustable with some transconductance devices

What input range is possible when using the tail current to program the OTA (i.e. after W/L fixed)?





$$g_m = \mu C_{OX}\frac{W}{L}V_{EB} == \sqrt{I_T}\sqrt{\mu C_{OX}\frac{W}{L}}$$

$$\mathbf{V_{dx}} = \pm\sqrt{\frac{\mathbf{2L}}{\mathbf{\mu C_{OX} W}}}\left(\sqrt{\mathbf{I_T}}\right)$$

- Input signal swing decreases linearly with decreases in g_m for fixed W/L
- One decade reduction in g_m results in one decade decrease in signal swing
- One decade reduction in g_m requires two decade decrease in I_T
- Though MOS OTA can have very good single swing with large V_{EB}, very limited tail current programmability with basic MOS OTA
- There are, however, other ways to program MOS OTA without big penalty in signal swing

Stay Safe and Stay Healthy !

End of Lecture 34